Strategic bidding in a Combinatorial Clock auction: The 700 MHz Canadian auction

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Using data provided by Industry Canada this paper seeks to understand what strategic bidding is in the 2014 700-MHz Canadian auction. The auction format is known as Combinatorial Clock. Its design seeks to induce truthful bidding, that is, a bidder is expected to bid in a way that at each round of the Clock Rounds stage her utility is maximised, and expected to reveal her true valuation for each bundle that make up her final bid in the Combinatorial stage.

The Combinatorial Clock auction is an innovative auction design which aims to overcome some of the problems evidenced in the application of the Simultaneous Ascending auction, a format that has been widely popular for assigning radio spectrum to commercial providers of mobile communications services over the last 20 years.

The paper uses publicly available data from the auction by Industry Canada. By focusing on the longest of the three auction stages, the Clock Rounds, it examines the evolution of several auction indexes as a means to devise an approach to understand bidders' strategic behaviour. It also uses a Revealed Preference theory framework to test whether bidders bidding was consistent with utility maximisation. When it is not, the paper analyses evidence to support claims of strategic behaviour.

1. The Canadian 700 MHz spectrum auction

In this section I will summarise the main aspects of the auction such as the product design, a short profile of the participants and the auction outcome.

The auction was organised in 7 spectrum blocks, 14 service areas or regions, and attracted 10 participants. Table 1 shows the spectrum blocks and the bands each block is defined over; two of the blocks (D and E) were sold as "unpaired" while the rest were offered in two paired sub-bands.

Block	Frequency	Pairing	MHz
A	698-704 MHz/728-734 MHz	paired	6+6 MHz
B	704-710 MHz/734-740 MHz	paired	6+6 MHz
C	710-716 MHz/740-746 MHz	paired	6+6 MHz
D	716-722 MHz	unpaired	6 MHz
E	722-728 MHz	unpaired	6 MHz
C1	777-782 MHz/746-751 MHz	paired	5+5 MHz
C2	782-787 MHz/751-756 MHz	paired	5+5 MHz

Table 1. Auction's offered products (blocks)

Canada's 700 MHz spectrum auction started on January 14, 2014, and was completed on February 13, 2014. There were 16 provisionally qualified bidders and 10 finally qualified bidders before the auction started. 8 bidders claimed wins over at least one licence. A total of about CAN $5.3 billion were raised, there were 108 rounds of bidding.

Industry Canada, the auctioneer, created products by grouping certain blocks such as B and C, D and E and, C1 and C2, while block A was sold as a single block; those four products combined with 14 country regions to offer a total of 56 products. In the allocation stage bidders were able to demand 0, 1 or 2 items, corresponding to 0, 1 or 2 licenses in each of products BC, DE and C1C2, and 0 or 1 in product A. A total of 98 (7 x 14) licenses was offered.

1.1. Who were the bidders?

The three Canadian wireless services incumbents and seven other companies participated in the auction, of which three were regional operators and the rest were small companies.

Rogers

Rogers Communication Inc. is Canada's largest voice and data telecommunications service provider. The wireless segment is one of the four main segments of Rogers Communications, which is Canada's only national carrier operating on the combined global standard GSM/HSPA+/LTE technology platforms.

According to Rogers 2013 Annual Report, their LTE network reached approximately 73% of Canada's population as of December 31, 2013. Using the spectrum licences held in 2013 (prior to the auction), the company had made a lot of effort and investments on their wireless network quality and coverage. At the end of 2013, Rogers had become the fastest LTE wireless network provider in Canada. In the 700 MHz auction Rogers expected to acquire the amount of spectrum which would enable them to expand their LTE coverage to rural areas and improve wireless services.

Rogers had expanded their services reach successfully in 2013 and became the first Canadian carrier to offer LTE roaming services for wireless customers travelling to the U.S. They signed an LTE roaming agreement with AT&T in September 2013. Rogers had an incentive to build a long-standing relationship with AT&T in order to offer the uninterrupted access to LTE roaming services to their customers.

TELUS

TELUS is one of the big three incumbents that provides a wide range of telecommunications products and services nationally. It based in the Vancouver, British Columbia area. TELUS launched LTE wireless service in February 2012 through a partnership with Bell. This 4G LTE wireless network covers 14 cities across Canada. TELUS markets include most of the major cities in Ontario. In 2013, TELUS expanded wireless network service to the Northwest Territories and the Yukon. Meanwhile, Bell and TELUS have signed a network sharing agreement for LTE services with the plan of only sharing the infrastructure, not the spectrum. According to TELUS business plan in 2013, the acquisition of 700 MHz spectrum was deemed important as the means to serve its rural customers. In light of the shared network history and partnership between Bell and TELUS, they entered the auction aspiring to win licenses in the same bands and in different service areas that would build a nationwide footprint.

Bell

Bell Media is owned by BCE, which is one of the big three wireless service providers in Canada. Bell had invested in new call centres in Québec and Ontario in 2013. And it added 25 new LTE markets reaching 80% of the Canadian population by the end of 2013. Bell had the intention to deploy broadband LTE network to rural communities, small towns and Canada's North by acquiring 700 MHz spectrum. According to Bell's annual report and the auction result, it invested $566 million in acquiring significant 700 MHz wireless spectrum across all service areas.

Videotron

Videotron Ltd is a subsidiary of Quebecor Inc., and emerged as a potential fourth national carrier. Videotron had a strong customer base in its home province Quebec. It had the expectation of developing outside its home market. In May 2013, Rogers and Videotron agreed on a long-term deal to join a shared LTE network in Ottawa (in the region of Ontario) and Quebec. The twenty-year LTE network sharing agreement between the largest national carrier Rogers and Quebecor Inc's division Videotron would help both of them in sustaining the favourable market condition in Quebec and the Ottawa region.

SaskTel

SaskTel is a large regional carrier in Saskatchewan. The company committed to building and developing the largest 4G wireless network infrastructure across the province in 2013. SaskTel maintained a strong focus on developing business in its home province. According to SaskTel's annual report in 2013, the company expanded the LTE wireless network successfully to the nine major centres across Saskatchewan. In light of the economic growth and greater need for wireless spectrum capacity, SaskTel understood the significance of acquiring 700 MHz spectrum.

MTS

MTS (Manitoba Telecom Services) is a large regional carrier in Manitoba. MTS was the first provider of 4G LTE wireless service in Manitoba, with an expectation of further deployment and constant evolving of its LTE wireless network. MTS announced its strategic wireless network sharing agreement with Rogers Wireless Partnership in June 2013. According to MTS's annual report in 2013, it launched domestic and international LTE roaming service in the fourth quarter of 2013. Its 97% wireless network coverage in Manitoba gave it local competitive advantages over other wireless network providers.

1.2. Auction results

The three largest Canadian operators won 31, 30 and 22 licenses, paying a combined 94% of total revenue (CAN $5.3 billion). One new player, Videotron, was able to build a large footprint as it won licenses in major provinces. The low frequency licenses were highly valued because the spectrum in such vicinity is especially attractive for the delivery of mobile services using next-generation wireless services, mainly LTE. Table 2 displays the overall results of the auction.

Licence Winners	# of Licences Won	Final Price	Total Population Covered
Feenix	1 Paired + 0 Unpaired	$284,000	107,215
MTS	1 Paired + 0 Unpaired	$8,772,072	1,206,968
Bragg	4 Paired + 0 Unpaired	$20,298,000	3,101,204
TELUS	16 Paired + 14 Unpaired	$1,142,953,484	33,475,915
Vidéotron	7 Paired + 0 Unpaired	$233,328,000	28,030,489
Bell	17 Paired + 14 Unpaired	$565,705,517	33,475,915
Sasktel	1 Paired + 0 Unpaired	$7,556,929	1,030,039
Rogers	22 Paired + 0 Unpaired	$3,291,738,000	33,368,700

Table 2. Canada's 700 MHz auction result

The level of competition for licenses as reflected by the final prices can be appreciated if such prices are compared to the initial (reserve) prices. Table 3 displays the intensity of competition via such relative prices of the licences, whereby the final price of the bundle won by a bidder is compared to the sum of initial prices (of licences in the bundle) set by Industry Canada (industry Canada, 2013) previous to auction start.

Bidder	Initial price	Final Price	Ratio
Feenix	$284,000	$284,000	1
MTS	$3,198,000	$8,772,072	2.74
Bragg	$8,218,000	$20,298,000	2.47
TELUS	$220,633,000	$1,142,953,484	5.18
Vidéotron	$120,969,000	$233,328,000	1.93
Bell	$200,024,000	$565,705,517	2.83
Sasktel	$2,755,000	$7,556,929	2.74
Rogers	$314,353,000	$3,291,738,000	10.47

Table 3. Relative prices of won bundles

The largest players display large differences in the final-price-to-initial-price ratio (column "Ratio" in Table 3). For instance, Rogers spent heavily as shown by a ratio of 10 which is by far the largest. Rogers won most of A licences and also created a nationwide footprint as it got at least one license from the BC product in all but the least valuable Canadian province. The ratio for the next incumbent, Telus, is also remarkably high. All others, including that of the third largest incumbent, are alike.

2. The auction rules

The 700 MHz auction in 2013 was organised in 56 products which corresponded to 14 geographical areas with 4 sets of bands in each area (three bands with two blocks each, known as *generic*, and one band with a single paired block). The essential CCA design consists of two main stages: the Allocation stage and the Assignment stage; in turn, the allocation stage is further divided in two: the Clock Rounds and the Supplementary Round. In the 700 MHz auction all bidders bid during the Clock Rounds and a fewer did it in the Supplementary round. Next, only those bidders who won in the Supplementary round at least one items of products (spectrum bands in a geographical area) where more than one license was being offered got to bid on specific targeted licenses in the Assignment stage. Figure 1 is a flowchart that illustrates the auction stages and conditions that allow a bidder to go from one stage to the next.

Previous to the auction the auctioneer used a point system to allocate "weights" to every unit of every product. Such weights provide an initial guide to the value of licenses and are used so that every bidder sets a number of points for herself to get started. Such number is called **initial eligibility**. As every round reveals individual, unitary prices for the 56 products, bidders respond by demanding a number of units of each product; the sum of points of those demanded units is called the bidder's "activity level". In the next round prices increase, though price increments in two different products are not necessarily the same.

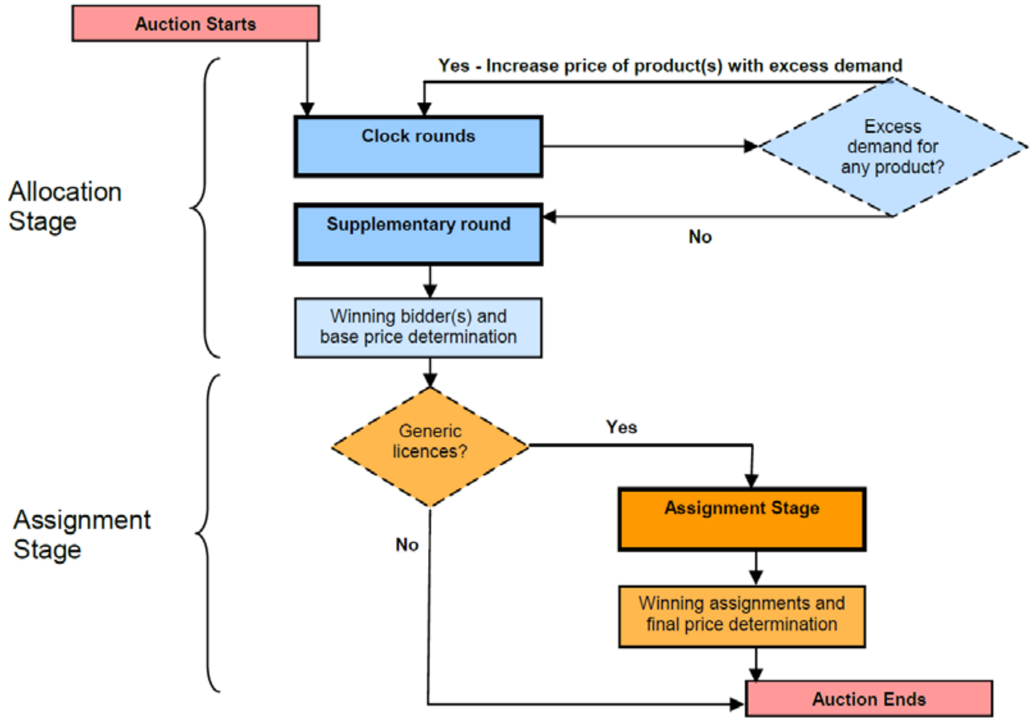


Figure 1. Combinatorial Clock auction flow chart (Source: Industry Canada, 2013)

On a given clock round a bidder's activity level value is the sum of points of those products included in the bidder's bid. The bidder's eligibility level is a measure of how "large" a bidder's bid can be at any round. Before the auction starts bidders choose their initial eligibility levels; then eligibility is either maintained if the bidder's activity level remains equal to the eligibility, or is decreased when the activity level is reduced. The round at which this happens is known as an **eligibility-reducing** round. The latter results in next round eligibility being set equal to the new, reduced activity level. Thus a bidder's activity should be equal or less than her eligibility; this is called the **activity rule**.

When a rounds closes, bidders are informed about the excess demand for each product. That is, the sum of all units demanded for each product less the number of units offered by the auctioneer.

Activity rules seek to induce bidders to bid on the items they really prefer forcing them to do so in a way that keeps them actively bidding throughout the auction. With such a rule in place the hideouts for bidders are minimised.

There is an exception to the activity rule: a bidder is allowed to bid on a bundle whose eligibility is greater than the bidder's current eligibility at any round if a certain condition is met. Such condition is known as the **Revealed Preference Condition** (RPC). The RPC allows a bidder to explore bundles whose eligibility exceeds the bidder's current eligibility. It may so happen that a bidder who is focused on a particular bundle during a number of rounds, with prices for the licenses in the bundle going up and his eligibility - possibly - going down, may have overlooked an opportunity to bid on an alternative group of licenses whose price is attractive – usually cheaper than the current price held by the bundle she is focused on - but with a higher eligibility than her current eligibility. With only the activity-based rule such bundle would be out of reach for a bidder; so it is important, in terms of efficiency, to allow bidders to bid on bundles for which they may have some preference even if they were not highly attractive at earlier times.

RPC imposes a condition on the relative prices of two bundles, namely, the new bundle q_t which breaks the eligibility condition and each of the bundles, say q_s for $s < t$, where an eligibility-reducing round occurred. Assuming that at round t a bidder has devised a bundle q_t as attractive, if the eligibility of q_t surpasses the bidder's current eligibility, the bidder can still place a valid bid for q_t if the following condition (RPC) holds:

For every eligibility-reducing round s ($s < t$),

$$x_t - x_s \leq y_t - y_s \qquad [1]$$

where $x_t = p_t \cdot q_t$, is the price of the new bundle q_t at round-t prices p_t
$x_s = p_s \cdot q_t$, is the price of the new bundle q_t at round-s prices p_s
$y_t = p_t \cdot q_s$, is the price of bundle q_s at round-t prices p_t
$y_s = p_s \cdot q_s$, is the price of bundle q_s at round-s prices p_s

The latter means that as long as the relative price of the new bundle is less than the relative price of each of the bundles demanded at eligibility-reducing rounds, the new bid is valid even if it violates the eligibility condition.

The following example illustrates the components of the activity rule using information of the Clock Rounds bids by a single bidder. The following is an excerpt of Videotron' bidding history the Clock Rounds from round 52 to 57. Though for each round the demand vector is available, it is not shown here.

Round	This round eligibility	This round activity	Next round eligibility	Bid (bundle) price
52	1304	980	980	304,899,000
53	980	782	782	202,172,000
54	782	782	782	212,284,000
55	782	782	782	222,902,000
56	782	782	782	234,052,000
57	782	1129	782	387,933,000

Using the notation as in [1] above, RPC can verified for round 57 when the eligibility is violated (1129 > 782). Verifying RPC has to be done for every eligibility-reducing round previous to 57, in this case round 52 (eligibility decreasing from 1304 to 980) and round 53 (from 980 to 782).

Inequality [1] is verified for t = 57 and s = 52 with

387,933,000 - 355,705,000 < 363,736,000 - 304,899,000

(values 355,705,000 and 304,899,000 calculated using individual item round prices published in Industry Canada 2015).

Inequality [1] is verified for t = 57 and s = 53 with

387,933,000 – 366,140,000 < 245,758,000 - 202,172,000

(values 366,140,000 and 245,758,000 calculated using individual item round prices published in Industry Canada 2015).

3. Looking deeper into auction data

Industry Canada (2013) provides a rich set of data for the 2014 700 MHz auction which allows us to look deeper into bidding patterns in the Clock Rounds, which is the main goal of this paper. Table 4 shows a summary of the Clock Rounds stage and the final allocation of licenses to winners.

For each bidder Table 3 displays the initial bid and the last bid in the Clock Rounds. It also includes the final outcome (after the assignment stage), which indicates how many licenses the bidder got and what services areas the licenses cover. In addition, the last columns show information about the bidder's initial eligibility and the activity level at the closing round.

One characteristic of this action is how much similar the final clock round bundle is to the actual won bundle for every bidder. In fact, the four smallest bidders got the exact same bundle each bid on in the last clock round. Only in one or two products was the last clock round bid different from the final outcome for Bell and Telus. The other winners got exactly what they bid for in the last round. In any case, the assignment stage was still needed to specifically allocate either B or C, and either C1 or C2 among the winners of exactly one license in the BC and C1C2 products.

	Round	NFL&L	NS&PEI	NB	EastQ	SouthQ	EastO	NorthQ	SouthO	NorthO	Mani	Sask	Alber	BC	Y&NWT	NFL&L	NS&PEI	NB	EastQ	SouthQ	EastO	NorthQ	SouthO	NorthO	Mani	Sask	Alber	BC	Y&NWT	NFL&L	NS&PEI	NB	EastQ	SouthQ	EastO	NorthQ	SouthO	NorthO	Mani	Sask	Alber	BC	Y&NWT	NFL&L	NS&PEI	NB	EastQ	SouthQ	EastO	NorthQ	SouthO	NorthO	Mani	Sask	Alber	BC	Y&NWT	Paired (# licenses)	Unpaired (# licenses)	Eligibility / Activity
		A Paired														B C Paired														D E Unpaired														C1 C2 Paired																
Feenix	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	198
	106	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	2
	final																																																								1	1	0	
																																																									C2			
MTS	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	2	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	2	290
	106	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	24
	final																																																		1					1	0	
																																																			C1							
Bragg	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	0	0	0	0	0	2	2	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	6	109
	106	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	0	0	0	0	0	4	0	62
	final																																											1	1	1						1						4	0	
																																												C1	C1	C1						C1								
Telus	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	28	28	3110
	106	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	1	1	1	1	1	0	0	0	2	2	2	0	0	0	0	2	2	2	2	1	1	1	0	0	0	1	1	1	0	0	0	0	0	15	14	1617
	final										1	1							1	1	1				1	1	1	1	1				2	2	2				2	2	2	2		1	1	1				1	1	1						16	14	
																			C	C	C				B	B	C	C	C															C2	C2	C2				C2	C2	C2								
Videotror	1	0	0	0	1	1	1	1	1	1	0	0	1	1	0	0	0	0	1	1	1	1	1	1	0	0	1	1	0	0	0	0	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	8	2456
	106	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	1	1	0	7	0	1113
	final																																														1	1	1	1	1				1	1		7	0	
																																															C1	C1	C1	C1	C1				C1	C1				
Novus	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	524
	62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	final																																																									0	0	
TBayTel	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	16
	48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	final																																																									0	0	
Bell	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	28	28	3110
	106	0	0	0	0	0	0	1	0	1	0	0	0	0	1	1	1	1	0	0	0	1	1	1	0	0	0	0	1	2	2	2	0	0	0	2	2	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	1	1	1	0	17	10	1495
	final							1		1					1	1	1	1				1	1	1					1	2	2	2				2	2	2					2				1	1	1				1	1	1	1		17	14	
																C	C	C				B	C	B					B																		C2	C2	C2				C2	C2	C2	C2				
SaskTel	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	109
	106	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	20
	final																																																			1				1	0	
																																																				C1						
Rogers	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	0	2	0	0	2	0	2	2	2	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	26	16	3110
	106	1	1	1	1	1	1	0	1	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	22	0	2374
	final	1	1	1	1	1	1		1				1	1		1	1	1	1	1	1	1	1	1	1	1	1	1																														22	0	
																B	B	B	B	B	B	C	B	C	C	C	B	B																																

Table 4. Initial and last bids in the Clock Rounds stage and final auction outcome.

Bidders in the CCA are faced with a large number of combinations (bundles) of items over which they define their preferences. In general, preferences depend on bidders' business plans, existing customer base and size of their current business.

As signals in lower spectrum in the 700 MHz band can travel longer distances and more easily penetrate buildings compared to upper 700 MHz band, the three lower 700 MHz paired blocks A, B and C turned out to be most attractive for bidders. Cramton (2014) notices that the combination of block A and block B are the most desired contiguous spectrum bands, and block C was the second choice for bidders, with the upper 700 MHz paired blocks C1 and C2 being the least attractive. Cramton's findings lend support to the claim that bidders followed a general preference ordering on different blocks. From the auction results we have identified some regular patterns that bidders applied in the Clock Rounds stage.

By looking at a bidder's Clock Rounds bidding history, we can infer that bidders followed a general preference ordering: A/B licenses were preferred to C, which in turn were preferred to C1/C2. As expected, blocks A, B and C were in great demand during the Clock Rounds stage. Most of the bidders bid on blocks A, B and C aggressively, especially when price had not risen high enough; and indeed, the final prices paid for such blocks were much higher than those of other blocks. From the Clock Rounds data we can further derive a preference model for each bidder. The following analysis will help us model bidders' own preferences derived from their clock rounds bidding history.

The figures below show each bidder's Clock Rounds bidding history for each block. The figures display the total number of units (blocks) of each band bid over by every bidder in each of the 106 rounds.

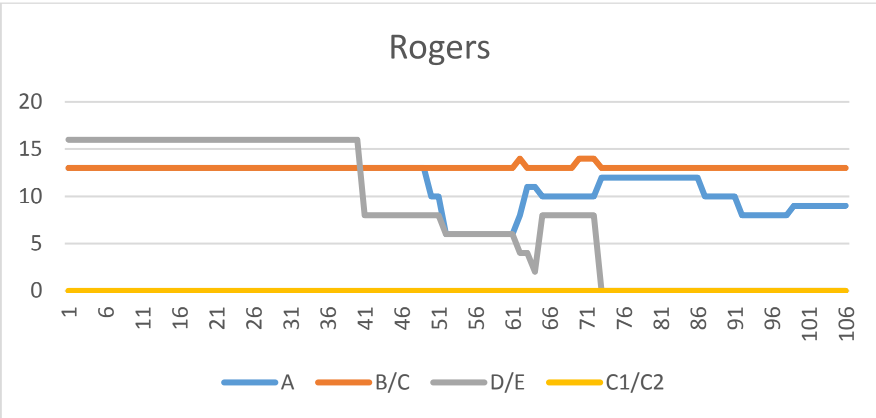


Figure 2. Rogers's number of licenses bid on for each product over the Clock Round stage

From Figure 2, it can be appreciated that Rogers had no interest whatsoever in any license in block C1\C2, whereas it maintained a high activity on licenses in the block B/C. One constraint imposed on the auction was a spectrum cap, which imposed on large carriers would mean a carrier could only gain a single block among the four prime blocks, namely B, C, C1 or C2. Rogers's next preferred block was block A, which is a paired block. Furthermore, the spectrum cap required that no bidder could gain more than two paired blocks. The data indicates that the third choice of Rogers was the unpaired block D/E. As a matter of fact, on round 73 Rogers gave up on all 8 items (two blocks in each of four regions) it had held on licences D/E reducing its demand to zero not to bid ever again.

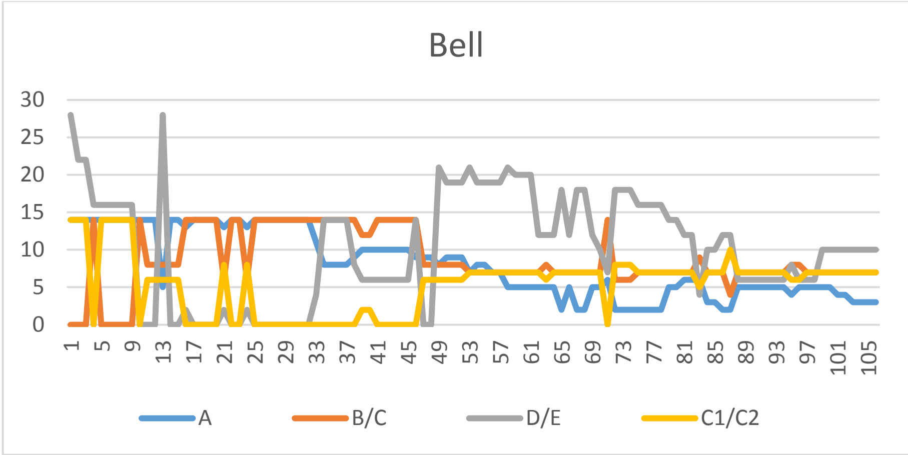


Figure 3. Bell's number of licenses bid on for each product over the Clock Round stage

Figure 3 depicts how aggressive were Bell's bids on block D/E, especially at the beginning of the auction and later after round 49. Bids on block A were also quite active for half of the Clock Rounds with a visible decline afterwards. All through the auction Bell alternatively bid the same number of items for block B/C and block C1/C2 in such a way that block B/C and block C1/C2 can be considered substitutes.

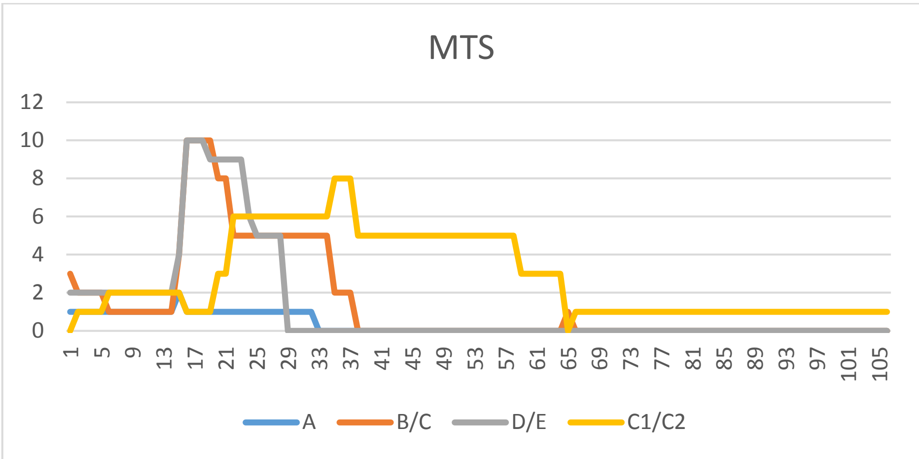
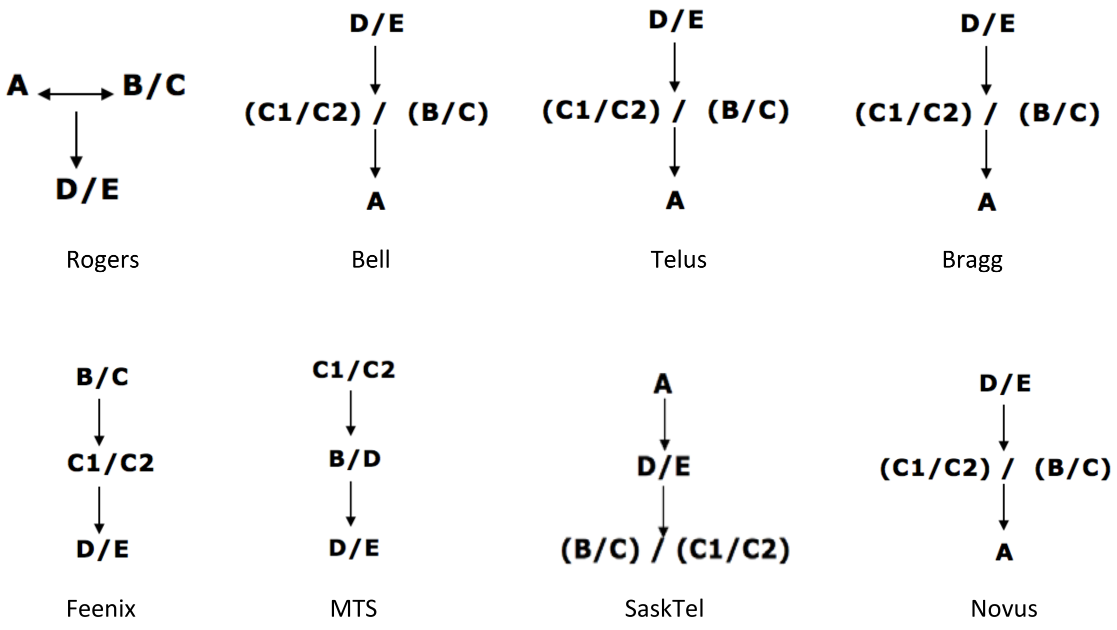


Figure 5. MTS's number of licenses bid on for each product over the Clock Round stage

Early on MTS started showing low interest in all blocks and none on C1/C2. As the auction progressed MTS started to ramp up its seeming interest in all blocks until it gave up block A (round 33), block B/C (round 39) and block D/E (round 29). Block C1/C2 proved to be the most attractive as MTS ended up winning one license, although only in one service area.

Tracing the aggregate number of demanded licenses throughout the rounds in the Clock Rounds stage for each product (block) reveals a preference structure that can be expressed as in the graphs below in Figure 6. In the graphs X → Y means that X is strictly preferred to Y, whereas X/Y means X and Y are substitutes; also X ←→Y means X and Y are complements.



Note: ↔ *denotes complement blocks;* **/** *denotes substitute blocks.*

Figure 6. Bidders' preference structure graphs as revealed by the aggregate number of licenses bid on for each product over the Clock Round stage

4. What aggregate data teach us about bidders' behaviour

Activity levels indicate how vigorous bidding may have been in a CCA. For instance a bidder with a given initial activity level that is substantially reduced at the end of the Clock Rounds stage may have encountered high prices that forced her to drop her activity drastically; on the other hand another bidder with a final activity level that did not fall in the same proportion as the former, may have had a somewhat easier time during the action that allowed her not to keep reducing her activity level, hence her eligibility, and so being able to stand stronger.

Table 5 provides a view of the vigour of competition found in the auction; it shows the ratios between final activity to initial activity and number of licenses earned to number of licenses demanded in round 1 for all winning bidders.

Licence Winners	Final activity level/ Initial activitylevel	# Licenses earned /# Licenses demanded in Round 1
Feenix	0.010	0.500
MTS	0.083	0.167
Bragg	0.569	0.400
TELUS	0.520	0.536
Vidéotron	0.453	0.292
Bell	0.481	0.554
Sasktel	0.183	1.000
Rogers	0.763	0.524

Table 5. Ratios of activity and total number of licenses to initial activity and number of licences won by each bidder.

Data on activity and eligibility through the rounds for a given bidder also allows us to define proxy measurements that describe the bidder's bidding behaviour. We use the activity level, the eligibility level and the number of licenses bid on by a bidder to propose three ratios (indexes) that describe the bidder's bidding strategy as the clock stage progresses. The index definition only applies to those rounds where the bidder is still active and applies to a given bidder.

For any round *t* in the clock rounds stage, let's define *e*(*t*), the **relative eligibility** index, as the ratio between the bidder's eligibility *E*(*t*) at *t* divided by the bidder's initial eligibility, *E*(1), that is

$$e(t) = \frac{E(t)}{E(1)}$$

For any round *t* in the clock rounds stage, let's define *a*(*t*), the **relative activity** index, as the ratio between the bidder's activity *A*(*t*) at *t* divided by the bidder's initial activity, *A*(1), that is

$$a(t) = \frac{A(t)}{A(1)}$$

For any round *t* in the clock rounds stage, let's define *q*(*t*), **relative license quantity** index, as the ratio between the number of blocks N(*t*) bid on at *t* divided by the number of initially bid-on blocks N(1), that is

$$q(t) = \frac{N(t)}{N(1)}$$

One example is useful to appreciate the information revealed by the indexes. Figure 7 displays *e*(*t*), *a*(*t*), and *q*(*t*) for Bell. For the whole duration of the auction Bell's activity was below or at its eligibility, hence its bids never violated the eligibility level.

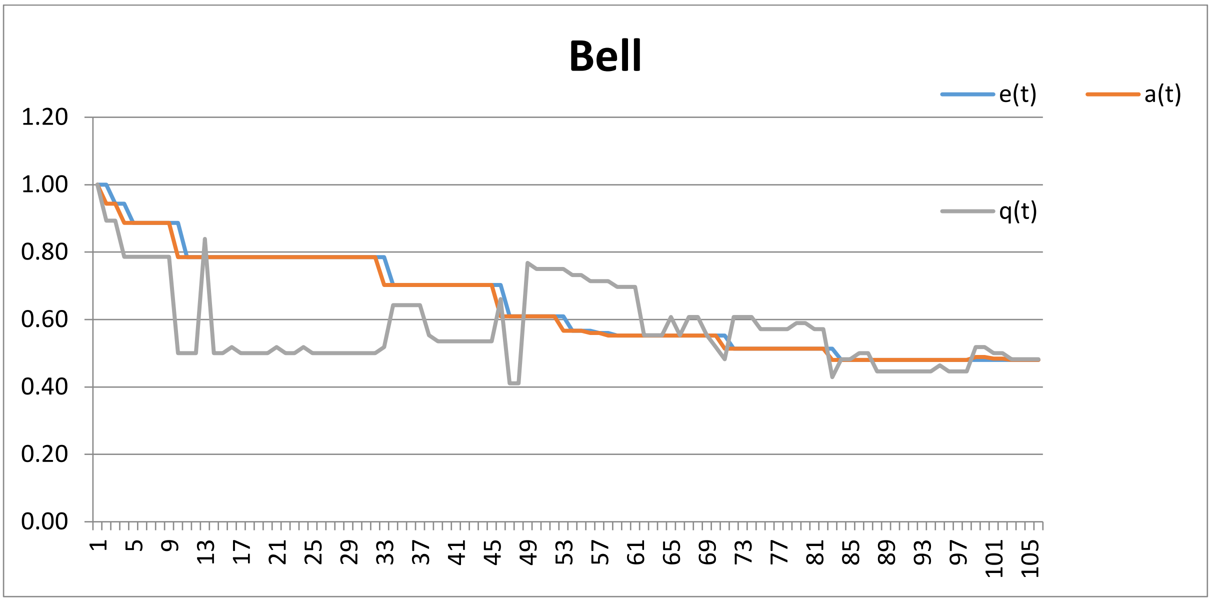


Figure 7. Relative activity, relative eligibility, relative license quantity and relative normalised price.

(Beltran, 2016) presents the indexes series for most bidders in the 700 MHz auction, pointing at specific behaviour patterns observed for different bidders, which reveal elements of strategic behaviour during the Clock Rounds.

5. Evidence that bidders do not bid in a straightforward manner

Some aspects of auction design are quite concerned with truthful revelation of information by the bidders. After all, if bidders reveal their true valuation for the spectrum licenses the auctioneer will be certain that the efficiency of the allocation will have been maximised. When an auction proceeds in rounds and prices are forming as bidding happens, bidders are also expected to bid in a way that is consistent with utility maximisation, that is, that at each round a bidder's bid maximises the difference between her valuation and the price of the bundle at posted individual licence prices. In practice a number of issues get in the way and not even bidders may find it easy to be certain that their bids are consistent with utility maximisation.

Bidding is such a way is known as **straightforward bidding**. Bidders are then said to be bidding rationally if they bid in a straightforward manner.

Recent work on the CCA (Knapek and Wambach, 2012), (Kroemer et al., 2016) reveal that often bidders do not necessarily bid in a way that is consistent with maximising their utility at every round. The evidence shown in (Kroemer et al., 2016) justifies our search for elements that may point at deviations from straightforward bidding that would be interpreted as strategic bidding.

The RPC allows bidders to discover prices of bundles that, in spite of having an eligibility level greater than the bidder's current eligibility, the bidder may find attractive even though the implied bidder's activity level turns out to be greater than the her eligibility. The discovered bundle is a substitute, to some degree, for bundles the bidder has already bid on. Such opportunity to bid does not go missing as the RPC allows the bidder to violate the eligibility rule as long as the relative price of the "discovered" bundle is less than the relative price of each of the bundles demanded at earlier eligibility-reducing rounds (see section 2).

RPC rests on the following observations (Ausubel et al., 2006) . If $v(q)$ represents the bidder's value for a bundle q, then for two rounds, s and t, a sincere bidder prefers q_s to q_t when prices are p_s , or

$$v(q_s) - p_s \cdot q_s \geq v(q_t) - p_s \cdot q_t$$

Likewise a sincere bidder prefers x_t to x_s when price are p_t , or

$$v(q_t) - p_t \cdot q_t \geq v(q_s) - p_t \cdot q_s$$

Combining the two inequalities, we get:

$$q_t \cdot (p_t - p_s) \leq q_s \cdot (p_t - p_s) \qquad [2]$$

Which means that, if $s < t$ the relative price of a "new" bundle q_t is less than the relative price of a bundle q_s that the bidder already bid on.

As mentioned above RPC is verified when eligibility is violated and only for certain rounds. In other words a departure from the central activity-based rule is allowed as long as the bidder proves to be bidding rationally in the sense dictated by RPC. No verifications of RPC are required for all previous rounds where eligibility was not violated.

The latter is what Bell's bidding behaviour reveals in the 700 MHz auction. Bell, one of the bigger players, bid in such a way that its relative activity index not only decreased monotonically over time, but never was larger than eligibility activity index, as can be verified by inspecting (Industry Canada, 2015).

On the other hand, Videotron spent a very large fraction of its time (rounds) in an eligibility-violated condition; Videotron seemed to be eagerly exploring prices of alternative bundles, possibly substitutes, a condition that could only be learned by using the RPC effectively (Beltran, 2016).

Interestingly enough, even though bidding in the CCA is heavily influenced by the application of the RPC, the rule only reveals a condition that logically follows the assumption that the bid revealed by the bidder at a given round and corresponding (aggregate) price are the best choices the bidder could have made at that round, as stated in [2].

However the converse condition implied by RPC would not be sufficient as a test that checks consistent or straightforward bidding. A known result, Afriat's theorem (Varian 2006), established a necessary and sufficient condition for a set of observations from choices made by a consumer to be consistent with utility maximisation, hence an expression of consumer rationality.

In order to understand the scope of Afriat's theorem let S be the set of observations, (p_i, q_i); i =1,. . ., N, where q_i is the bundle demanded at round i at prices p_i, and N is the last round.

For two observations i and j, if $p_i \cdot q_i \geq p_i \cdot q_j$ then it is said that q_i is **directly revealed preferred** to q_j and is denoted as $q_i R_0 q_j$. When the inequality is strict, that is $p_i \cdot q_i > p_i \cdot q_j$, then it is said that q_i is **strictly directly revealed preferred** to q_j and is denoted as $q_i P_0 q_j$.

If there's a sequence of indexes $k, u_1, u_2, \ldots, u_m, h$ such that

$$p_k \cdot q_k \geq p_k \cdot q_{u1} \; ; p_{u1} \cdot q_{u1} \geq p_{u1} \cdot q_{u2} \; ; \ldots ; p_{um} \cdot q_{um} \geq p_{um} \cdot q_h$$

or, equivalently,

$$q_k R_0 q_{u1} \; ; \quad q_{u1} R_0 q_{u2} \quad ; \ldots ; \quad q_{um} R_0 q_h$$

it is denoted as $q_k R q_h$. The set of all q_k, q_h such that $q_k R q_h$ is the **transitive closure** of R_0 .

Furthermore, the data set S satisfies the **Generalised Axiom of Revealed Preference** (GARP) if for each pair of bundles q_i, q_j the following holds:

If $q_i R q_j$ then it is not the case that $q_j P_0 q_i$

Equivalently, when $p_i \cdot q_i \geq p_i \cdot q_j$ then $p_j \cdot q_j \leq p_j \cdot q_i$

The theorem can now be stated:

Afriat's theorem (1967): A finite set of data is consistent with utility maximization if and only if it satisfies GARP.

Varian (2006) presents several equivalent statements for the theorem. The one used here allows us to use GARP as a test for consistency in bidding.

GARP is a sharp test. For an observation point such that $q_i R q_j$, if GARP condition is not satisfied then the theorem invalidates the whole set S. Cherchye et al. (2014) introduce a refinement of GARP to account for "how much" the condition is violated, or, in other words in the context of the present work, how far from "consistent bidding" a bidder may be.

In order to achieve that, first of all, let's work with normalized prices. All products $p_i \cdot q_i$ are normalised to 1, that is, $p_i \cdot q_i = 1$ for all i.

Let $0 \leq e \leq 1$. For every pair (p_i, q_i) of observed quantities and prices consider those q_j, $(j \neq i)$ such that

$$e * (p_i \cdot q_i) = e * 1 \geq p_i \cdot q_j \qquad [3]$$

Such inequality defines a relation $R_o(e)$ in Q x Q, (where Q is the set of all bundles) such that $(q_i, q_j) \in R_o(e)$ if and only if [3] holds. We can then focus on, $R(e)$, the transitive closure of $R_o(e)$.

A modification of GARP is now necessary. It is said that S satisfies GARP(e) if for each pair of bundles (q_i, q_j), $j \neq i$, the following holds:

If $q_i R(e) q_j$ then it is **not** the case that $q_i P_o(e) q_j$

where $q_i P_o(e) q_j$ is the expression: $e > p_j \cdot q_i$ (because we normalised the products $p_i \cdot q_i$)

Kroemer et al. (2016) propose an approach based on testing a stronger variation of GARP, thus exhibiting a measurement of "how consistent" bidding was in the Canadian 700 MHz auction, as quantified by the maximum value of e that would make a bidder pass a GARP(e)-based test. The maximum is well defined because if e' is such that GARP(e') is satisfied by the set S, then GARP(e'') is also satisfied by S for all $e'' < e'$.

Figure 8 exhibits an algorithm, based on Kroemer at al. (2016) and Cherchye et al. (2014), that finds the maximum value e^* for which the observations in S satisfy GARP(e^*). The algorithm uses Warshall's method to find the transitive closure of an order relation R, which is used to test GARP(e).

Algorithm to find the maximum value e^* for which the observations in S satisfy GARP(e^*)

Entries: S, set of observations, (P_i, q_i); i =1,. . ., N, where q_i is the bundle demanded at round i at prices P_i, and N is the last round.

Output: e^*, maximum value of e in [0, 1] for which all observations satisfy GARP(e)

For each round i (= 1, …, N) normalize prices P_i as follows:

$$\text{Let } p_i^1 = \frac{P_i^1}{p_i q_i} \text{ where } P_i = [P_i^1, P_i^2, \cdots, P_i^n]$$

Construct an array A of all values $p_i \cdot q_j \leq 1$ for $i \neq j$

Sort these values in ascending order

While A has more than one element

Let x be the median value in A.

Test GARP(x).

If GARP(x) is satisfied then

Remove all values lower than or equal to x from A

Else

Remove all higher values

End While

Let $e^* = x$.

Figure 8. Algorithm to determine an index for a bidder's rational bidding.

After normalising the prices observed in each round in such a way that, $p_i \cdot q_i = 1$ for all *i*, an array of values is obtained with all combinations of prices p_i and quantity vectors q_j such that $p_i \cdot q_j \leq 1$. Next, a search is done recurrently for the median value in A, removing any value *x*′ less than *x* because if *x* satisfies GARP(*x*), then any such value *x*′ also satisfies GARP(*x*′), or else, as GARP(*x*) is not satisfied, remove any value *x*′′ greater than x because any such *x*′′ does not to satisfy GARP(*x*′′).

6. Conclusions

This paper proposes an approach to understanding strategic bidding in the 2014 700-MHz Canadian auction. If focuses solely on the longest of the three auction stages: the Clock Rounds stage. Several flaws and issues that have arisen over time, mainly with the Simultaneous Ascending auction, have led to changes in the preferred way to sell spectrum via auctions. Such changes are reflected in the rules and conditions on bidding that were introduced to the CCA.

Our approach started with a qualitative assessment of substitution and complementary across products; by means of a keeping track of the number of licenses (services areas) receiving bids in a particular band and comparing such numbers across the 4 products (bands) preference relations were built for every bidder. Next, our interest shifted to using aggregate auction data to shed light on how bidding unfolded; we measure auction bidding activity and compare it among bidders. Aggregation is done via indexes that reveal differences in the way bidders use the auction rules regarding activity and eligibility levels.

Finally a Revealed Preference theory framework is proposed which allow for testing the extent to which a bidder’s bidding is consistent with utility maximisation. Revealed Preference is a non-parametric approach to observing rational behaviour that uses revealed demand data and axioms of rational preference.

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